SHEPHERD OPEN MATH CONTEST 2007

1. All students are eligible to take part in this contest.

2. The duration of the contest is two weeks between December 3rd and December 17th, 2007. Students are allowed to consult textbooks and the internet but should not be allowed to get help from mathematicians or other persons.

3. Please enclose the following signed and dated statement. "I certify that all the work that I am submitting is original and I did not receive anybody's assistance."

4. The solutions must be mailed (using regular mail and postmarked by December 15th latest) to 217, Byrd Science Center, School of Natural Sciences and Mathematics, Shepherd University, Shepherdstown, WV - 25443. The name and school of the student should be legible and the words "SHEPHERD OPEN MATH CONTEST" should appear on top of all sheets. The solutions should be handwritten neatly on clean white sheets.

5. The first prize is $500 cash, the second prize is $300, third prize is $200 and an honorable mention would be considered for $50. We reserve the right not to award any prizes if the quality of the solution does not meet our expectations. The winners will be invited to Shepherd University and recognized at an awards ceremony.

6. The winners will be announced in the middle of January. We are planning to make this an annual event.

1. Simplify

$$\frac{2\left(\frac{a}{-a+b+c+d}+\frac{b}{a-b+c+d}+\frac{c}{a+b-c+d}+\frac{d}{a+b+c-d}\right)+4}{\left(\frac{1}{-a+b+c+d}+\frac{1}{a-b+c+d}+\frac{1}{a+b-c+d}+\frac{1}{a+b+c-d}\right)}$$

2. Find the set of all points in the plane whose perpendicular projections fall on the sides of a given triangle (not on their extensions). Carefully distinguish the cases of acute, right angled and obtuse triangles.

3. (a) What conditions must a, b, c satisfy so that $P(x) = ax^2 + bx + c$ has two distinct real roots both in the interval $(-1, 1)$.

   (b) If a, b, c are all integers, all less than or equal to 3 in absolute value, how many polynomials $P(x) = ax^2 + bx + c$ exist with roots in the interval $(-1, 1)$.