Simplify the following algebraic expressions assuming that the denominators are never zero. The following problems are a challenge to the students in the CME department. They are due the Monday after spring break i.e. (March 26th). Those of you who are able to simplify the expressions perfectly will receive a lunch treat from the department.

1.
$$\frac{\left(\frac{x^3-y^3}{x^2+y^2}\right)\left(\frac{x^2-y^2}{x^3+y^3}\right)\left(\frac{1}{x^2}+\frac{1}{y^2}\right)}{\left(\frac{(x+y)^2-xy}{(x-y)^2+xy}\right)\left(\frac{1}{y}-\frac{1}{x}\right)}$$

2.
$$\frac{a}{1-a^{-1}}+\frac{1-a^{-1}}{\left(\frac{1}{1-a}\right)}+\frac{\left(\frac{1}{1-a}\right)}{a}$$

3.
$$\left(\frac{x^{-\frac{1}{6}}-\left(\frac{5}{y^{\frac{1}{6}}}\right)}{x^{-\frac{1}{3}}-y^{-\frac{1}{3}}}-5\cdot\frac{x^{-\frac{1}{6}}-y^{-\frac{1}{6}}}{x^{-\frac{1}{3}}-\sqrt[3]{y^{-1}}}\right)^{-1}\left(6\cdot\frac{\sqrt[6]{x}}{(\sqrt[3]{x}-\sqrt[3]{y})}\right)$$

4. Let
$$E=\frac{x^2+ax-b}{x^2-bx+c}.$$
Compute E for
$$x=-\sqrt{\frac{b^2-ac}{a+b}}.$$

5.
$$\left(\frac{a\cdot\frac{b-c}{b+c}+b\cdot\frac{c-a}{c+a}+c\cdot\frac{a-b}{a+b}}{\frac{b-c}{b+c}+\frac{c-a}{c+a}+\frac{a-b}{a+b}}\right)$$

(Aside: A cyclic sum is a summation where the next terms are obtained by replacing $a\to b, b\to c, c\to a$ in that order until the first time is obtained again. Using this new terminology, the above problem is one in which the numerator and denominator of the above expression are the cyclic sums
$$\sum_c a\cdot\frac{b-c}{b+c},\ \sum_c\frac{a-b}{a+b},$$
respectively.)